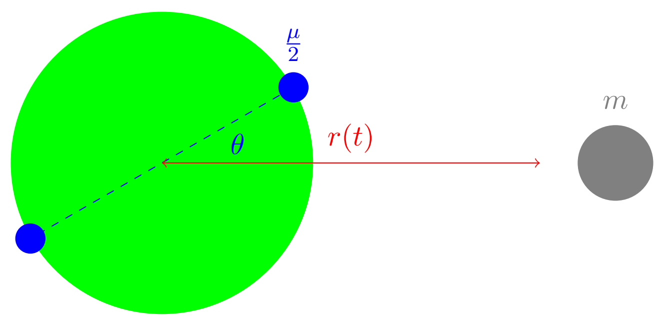
# The Earth-Moon Distance

The distance between the Earth and the Moon is increasing over time, due to the "drag" on the Earth's tides caused by the moon. In this problem, we will consider a very simple model for this effect.

Let $M \approx 6 \times 10^{24}$ kg be the mass of the Earth, and $m \approx 7 \times 10^{22}$ kg the mass of the moon, let $R \approx 6 \times 10^6$ m be the radius of the Earth, let $\omega_0 \approx 7 \times 10^{-5}$ s$^{-1}$ be the angular velocity of the Earth today and $\omega(t)$ be this angular velocity at a later time, let $r_0 \approx 3.8 \times 10^8$ m be the present day distance between the two objects, and let $r(t)$ be the distance between the Earth and moon at a time $t$ from today. You may also use that Newton's gravitational constant is $G \approx 7 \times 10^{-11}$ m$^3$/kg $\cdot$ s$^2$. Assume that the Earth is approximately a sphere of constant mass density. Finally, approximate that the axis of the Earth's rotation is identical to the axis of the Moon's orbit around the Earth, which you can approximate is a circle.

(a) Using angular momentum conservation and assuming that the Moon orbits around an approximately stationary Earth according to Newtonian gravity, find a relationship between $\dot{r}$ and $\dot{\omega}$.

To find $\dot{\omega}$, we need to incorporate what the Moon does to Earth's tides. We will model the tides as approximately 2 point masses of mass $\mu$, oriented on opposite sides of the Earth and making a relative angle $\theta(r)$ to the Moon, as shown below:




The gravitational forces acting on these tidal "point masses" will cause a torque on the Earth.

(b) Find $\dot{\omega}$ in terms of $\theta(r)$, $\mu$, $G$, $m$, $M$, $R$ and $r$. Approximate that $r \ll R$ and $\theta \ll 1$ and keep only the lowest order nontrivial term in your expression. Show that it is negative.

(c) Using the present day values for each of the parameters, along with the approximation $\mu \approx 10^{16}$ kg and $\theta \approx 0.01$, evaluate $\dot{r}$ and $\dot{\omega}$.

(d) Without an expression for $\theta(r)$, we cannot use this model to find the final distance $r_\infty$ between the Earth and Moon. A crude upper bound for $r_\infty$ comes from angular momentum conservation. If the Earth stops rotating about its axis, what would be $r_\infty$? Find an expression in terms of variables, and then evaluate the expression plugging in for numerical values.

(e) Show that the rotational energies of both the Earth and the moon is *not* conserved. How much energy is gained or lost? Where do you think it came from, or goes?