Finite Time Blow Up in Newton's Law

Consider a point particle of mass m in the potential well

$$V(x) = -V_0 x^a,$$

where $a > 0$ is some constant. At time $t = 0$, the particle is at $x = 0$, with some small velocity $v_0 > 0$.

(a) Using energy conservation, reduce the dynamics to a simple first order differential equation.

(b) Although this equation cannot be exactly solved, describe the dynamics at late time. In particular, show that for $a > a_c$, where a_c is some critical value you should determine, the solution $x(t_0) = \infty$ at some finite time t_0. What is the asymptotic behavior of $x(t)$ for $a = a_c$, and $a < a_c$?

(c) If $a > a_c$, estimate the time t_0, given the initial conditions at $x = 0$. Be sure to consider both the cases when v_0 is large, and when v_0 is small.

(d) One way such a potential can be set up is in a rotating reference frame. For example, consider a particle on a rotating circular turntable, constrianed to only move in the radial direction (about the axis of rotation). If we are in a reference frame which is rotating at constant angular velocity ω, determine the values of V_0 and a. What is the late time dynamics in this case? Explain how the answer you find is consistent with conservation of energy in an inertial reference frame.