# Hourglass on a Scale

A classic physics problem is that the weight of an hourglass, as observed by a scale, is "independent" of whether all of the sand is at rest at the bottom of the hourglass, or whether there is sand falling in the hourglass. The reason for this is that the weight of the sand that is falling, which is not counted in the weight seen on the scale, is offset by the force of the sand hitting the bottom of the hourglass.

In fact, *this is wrong*, as we'll show in this problem, for a finite sized hourglass. Consider the hourglass shown below, with the top and bottom having a constant cross sectional area A, and a small intermediate neck, in between. Let M be the total mass of both the sand and the hourglass. Assume the rate of mass flow, α, through the hourglass, in steady-state, when sand of mass density ρ flows from top to bottom, is independent of the height of sand in the top of the glass.[1]:
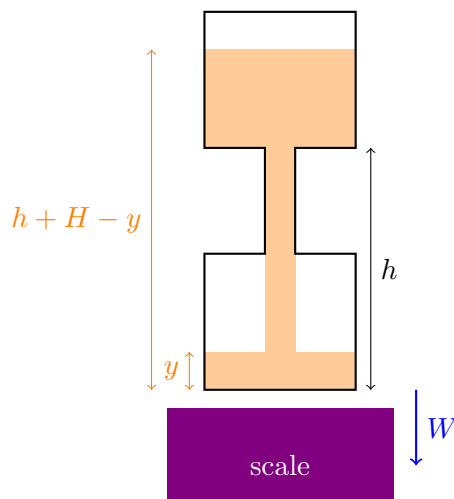
(a) Find an expression for the vertical position of the center of mass, when the height of the sand in the bottom of the hourglass is y, and when the vertical height of the sand in the top of the hourglass is $H - y$. You can neglect the sand that is falling through the hourglass, as this is negligible.

(b) Find the velocity and acceleration of the center of mass of the sand.

(c) In the intermediate steady-state when sand is falling from the top, show that the weight observed on the scale is

$$W_{\text{int}} = Mg + \frac{2\alpha^2}{\rho A}.$$

where M is the total mass of the sand and hourglass.

(d) Physically, where does this extra force come from? Why does the argument presented at the beginning of this problem fail?

(e) Carefully sketch qualitatively the weight $W(t)$ observed on the scale as a function of time, and explain the physics. Be careful about early and late times!

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[1]This would not be true if the sand at the top of the glass was a fluid. In this case, the mass flow rate would decrease as the height of the fluid decreased.