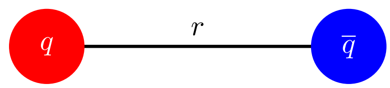
classical mechanics → Newtonian mechanics ★★

String Model of Mesons

Protons, neutrons and many other fundamental particles are made up of subparticles called quarks. In addition to a quark, there is an antiquark particle. Quark-antiquark pairs form particles called mesons, which are unstable high energy particles produced by giant particle colliders such as the LHC.

There is a simple model for the internal structure of a meson, where the meson consists of a quark q and an antiquark $\overline{q}$, connected by a string[1] of radius r and constant tension T.



(a) What is the energy E stored in the string, if the two particles are stretched a distance r apart?

(b) $E = mc^2$ is Einstein's famous formula relating mass to energy. If the mass of a meson is about $m \approx 10^{-27}$ kg, and the speed of light $c = 3 \times 10^8$ m/s, and the distance scale between the two ends of the string is about $r \approx 10^{-15}$ m, estimate the tension T. All of the energy is this problem is the potential energy stored in the string. Compare it to a real world force and comment.

Actually, what we did is not quite correct. There are quantum stringy corrections to the mass formula. Let E_0 be the energy you found in part (a). It turns out that this must be corrected to

$$E^2 = E_0^2 - \frac{\pi}{6}\hbar c T.$$

The constant c is the speed of light. The constant $\hbar \approx 10^{-34}$ J · s is a quantum mechanical constant.

(c) Find $m(r)$, the mass as a function of length r of the string, including the quantum correction given above. You do not need to plug in for the numbers yet! Sketch your result and comment on what happens, both at large r and small r.

(d) For large r, what is the leading quantum correction to the mass? Is this important at the radius r we suggested in part (b)? Interestingly, this formula for $m(r)$ (more properly, $E(r)$) is found approximately in very complicated computer simulations of quarks.

(e) As $r \to 0$, you should find that $E^2 < 0$! This means that our particle is what is called a *tachyon* (made famous by sci-fi!) – it is a particle whose mass is an imaginary number. These particles are unstable. In fact, we can estimate the lifetime τ of a tachyon by

$$\tau \approx \frac{\hbar}{\sqrt{-E^2}}.$$

Estimate the lifetime of a meson. Compare to known lifetimes of about 10^{-9} s: does this model explain the lifetime of a meson? Comment on the answer.

[1] This strange model is in fact, the origin of string theory! Now we think that string theory is much more powerful than this, however.