Penning Trap

A device called a Penning trap is used to trap charged particles, such as protons, in a stable fashion for very long periods of time. This problem will show you how such a device works, at least at the classical level.

Consider a particle of charge q and mass m, placed in the scalar potential

$$\varphi = \alpha\left(z^2 - \frac{x^2+y^2}{2}\right)$$

for α some positive constant and the vector potential

$$\mathbf{A} = \frac{B}{2}\hat{\mathbf{z}} \times \mathbf{x}.$$

Also in this problem, you may find it convenient to define a pair of frequencies

$$\omega \equiv \frac{qB}{m},$$
$$\Omega \equiv \sqrt{\frac{q\alpha}{m}}.$$

(a) What are the electric and magnetic fields due to these potentials?

(b) Write down the equations of motion for the charged particle, in cylindrical coordinates. It may be helpful to use the Lagrangian formulation of mechanics to do this.

(c) Show that the motion in the z direction decouples from the planar motion. What type of motion is in the z direction?

(d) Now, turn to the planar motion. Show that the motion can be reduced to central force motion, and find the effective potential.

(e) Show that the qualitative trajectory of the charged particle qualitatively changes depending on the relative values of Ω and ω. Describe how we should choose these values relatively in order to stably trap a charged particle.

(f) Assuming that the motion in the radial direction consists of small oscillations, sketch a trajectory of the charged particle in the plane.