# Correlator Equations of Motion

Consider some quantum field theory given by Euclidean action $S[\phi]$ (for some single quantum field $\phi$), with partition function given by

$$Z = \int \mathrm{D}\phi \, \mathrm{e}^{-S[\phi]}.$$

Let $F[\phi]$ be some arbitrary functional of $\phi$.

(a) Show that by shifting $\phi \to \phi + \epsilon$ for arbitrary infinitesimal and constant $\epsilon(x)$, that

$$\left\langle \frac{\delta F}{\delta \phi(x)} \right\rangle = \left\langle F \frac{\delta S}{\delta \phi(x)} \right\rangle.$$

In general, this is an equation of motion relating various correlators, as you will now show with a simple example.

(b) For simplicity, suppose that $S$ is of the form

$$S[\phi] = \int \mathrm{d}x \, \frac{1}{2}\phi(x)\mathcal{P}\phi(x)$$

with $\mathcal{P}$ some differential operator. Use $F = \phi(y)$ and show that the propagator of the $\phi$ field is a Green's function for the operator $\mathcal{P}$.

The technique described above generalizes straightforwardly to more complicated theories, as should be clear from the derivation.