Feynman Rules with Long Range Interactions

Consider the quantum field theory of a single real scalar field $\phi(x)$, with $x \in \mathbb{R}^d$, with a "bare" action

$$S_0[\phi] = \int \mathrm{d}^d x \left[\frac{1}{2}(\partial\phi)^2 + \frac{m^2}{2}\phi^2 \right].$$

Suppose that the action is actually $S = S_0 + S_{\mathrm{LRI}}$, where

$$S_{\mathrm{LRI}} = \frac{1}{2}\int \mathrm{d}^d x \, \mathrm{d}^d y \; J(|x-y|)\phi(x)\phi(y).$$

This adds a long range interaction term to the action.

(a) Explain how the long range interaction changes the propagator for this field ϕ.

(b) Discuss what happens semi-quantitatively (i.e., don't worry about overall constants) when

$$J(r) = \frac{C}{r^A}.$$

Explain how there is a critical A_{c} such that for $A < A_{\mathrm{c}}$, the long range interaction plays a dominant role in the dynamics of the theory.

(c) Discuss what happens semi-quantitatively when

$$J(r) = J_0 \mathrm{e}^{-r/\xi}.$$