Flux Line Depinning

Columnar defects can be formed in a superconductor by heavy ion irradiation. When a columnar defect forms, magnetic flux lines through the superconductor[1] typically become trapped around these defects. Experimentalists have found this technique useful because it allows them to increase the critical current carried by the superconductor. This problem will give a very simple mode of the quantum process of flux line depinning.

We can crudely approximate that the flux line is like a "quantum elastic string" vibrating along the columnar defect. This is essentially because twisting and turning the magnetic flux line should, to lowest order, require energy, and the quadratic approximation is the simplest. Let us denote by $u(z, \tau)$ the displacement of the string in a single dimension perpendicular to the defect[2] where τ is imaginary time. We can express the (Euclidean) action as

$$S = \int \mathrm{d}z\mathrm{d}\tau \left[\frac{1}{2}\rho\left(\frac{\partial u}{\partial \tau}\right)^2 + \frac{1}{2}\sigma\left(\frac{\partial u}{\partial z}\right)^2 + V(u)\right]$$

where $V(u)$ is some potential energy function.

Now, suppose we apply an external force to this flux line. This corresponds to adding an external contribution to the action

$$S_{\text{ext}} = -F\int \mathrm{d}z\mathrm{d}\tau\ u.$$

(a) Rescale z and τ so that the action appears symmetric in z and τ.

(b) Find the saddle point equation of motion for u.

(c) As before, we are looking for instanton solutions. You can still assume radial symmetry, with $u = u(r)$, where $r = \sqrt{z^2 + \tau^2}$. We can approximate that

$$u(r) = \begin{cases} u_0(r) & r < R \\ 0 & r \geq R \end{cases}$$

i.e. for $r < R$, the flux line is trapped to the defect, but for $r > R$ it is freed, and thus has only free dynamics. Using the thin wall approximation that the variation of the potential is negligible on the scale of "bubble" formation, determine the instanton forms of $u_0(r)$, depending on R. (Assume u is continuous!) Also, explain why we had to take the approximate form for u which we did.

(d) Approximate $V \approx V_0$, as $V(u)$ should be roughly constant over the scale of bubble formation. Finding the extremum of $S_{\text{inst}}(R)$, show that the rate at which the flux lines can depin from the defects is approximately

$$\Gamma \sim \exp\left[-\frac{4\pi\sqrt{\sigma\rho}{V_0}^2}{F^2}\right].$$

[1] Note this implies the superconductor is type II.

[2] Consideration of the second perpendicular dimension is not important for the purposes of this problem, which are strictly qualitative.