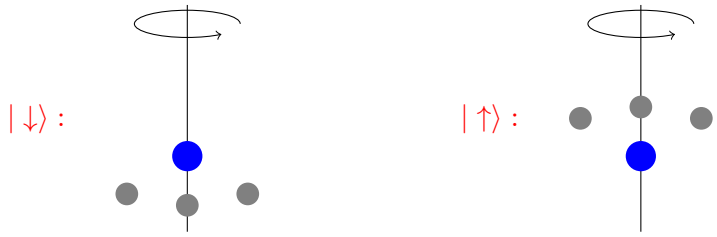
Ammonia

Ammonia (NH_3) is a highly polar molecule, with dipole moment $p = 4.7 \times 10^{-30}$ C · m. Ammonia undergoes quantum oscillations between the two phases of up and down, where the orientation of the dipole moment reverses, while the overall angular orientation of the rotating molecule is preserved. Let us consider the ammonia molecule in the presence of an electric field of strength $\mathcal{E}$ – we approximate that the ammonia molecule's axis of rotation is aligned with this field. The quantum dynamics of this system can be modeled by the Hamiltonian

$$H = \begin{pmatrix} -p\mathcal{E} & -\eta \\ -\eta & p\mathcal{E} \end{pmatrix}.$$

In this problem, use the numerical parameter $\eta = 4.1 \times 10^{-20}$ J.



(a) Find the eigenvalues of the Hamiltonian. Comment on what happens in the limit of small $\mathcal{E}$ and large $\mathcal{E}$.

(b) Find the eigenvectors of the Hamiltonian. Comment on what happens in the limit of small $\mathcal{E}$ and large $\mathcal{E}$.

(c) For small electric fields, find the leading order correction to the splittings in energies due to the electric field.

(d) Find an expression for the polarizability α of ammonia, in terms of p and η. Plug in for the numerical values of p and η. For what electric fields will this expression break down – are these large or small fields?

(e) Do you think ammonia has a small or large dielectric constant?