Neutrino Oscillation

Neutrinos are nearly massless and non-interacting fermions produced in particle interactions. They come in three "flavors": $|e\rangle$, $|\mu\rangle$, $|\tau\rangle$, corresponding to the electron, muon and tau (the three types of leptons, a type of particle). In the Standard Model, they are predicted to have no mass. However, it is now known that neutrinos have very small, but non-zero, mass.

As a consequence of this mass, neutrinos can oscillate between differing states; i.e., there is no reason for $|e\rangle$, $|\mu\rangle$ and $|\tau\rangle$ to be the eigenstates of the Hamiltonian! Although this certainly is not intuitive, there is no reason a priori not to allow for this. For simplicity, let's ignore the $|\tau\rangle$ state. We are free to write the two energy eigenstates of the Hamiltonian as $|1\rangle$ and $|2\rangle$:

$$H = E_1|1\rangle\langle 1| + E_2|2\rangle\langle 2|.$$

The "particle" states are given by a linear combination:

$$|e\rangle = \cos\theta|1\rangle - \sin\theta|2\rangle$$
$$|\mu\rangle = \sin\theta|1\rangle + \cos\theta|2\rangle.$$

θ is an unknown parameter that could be measured experimentally. Assume that the neutrinos do not change their momentum when undergoing oscillations between states, although their energy will change due to the change in mass. Furthermore, assume that the neutrons are ultrarelativistic, but not perfectly relativistic (i.e. $E \neq pc$). Let m_1 and m_2 be the masses of the Hamiltonian eigenstates.

(a) If a neutrino starts in the $|e\rangle$ state, and travels a distance d, show that the probability it is still in the $|e\rangle$ state is given by roughly

$$|\langle e|\psi(d)\rangle|^2 \approx 1 - \sin^2 2\theta \sin^2\left[\frac{({m_2}^2 - {m_1}^2)c^3}{4\hbar E}d\right].$$

In the sun, neutrinos are formed by the fusion of hydrogen atoms; they initially start in the $|e\rangle$ state. The neutrinos are produced throughout the sun, and not at any one point.

(b) What is the rough order of the minimum radius of the sun for which the neutrino oscillations will be "washed out": i.e., the probability of finding a neutrino in the state $|e\rangle$ is independent of position?

(c) Assuming the oscillations are washed out, in terms of θ, what is the probability of finding an neutrino in the state $|e\rangle$?

(d) We only see about 33% of the electron flavor neutrinos that solar models would predict. Can this phenomenon be explained by the existence of only two flavors, or do we require the existence of a third?