PT Transition in 2 State System

One of the classical postulates of quantum mechanics is that the Hamiltonian is Hermitian – i.e., $H = H^\dagger$. This ensures that the Hamiltonian's eigenvalues, the allowed energies of the system, are real.

However, there are more generic classes of Hamiltonians which are *not Hermitian*, yet nonetheless have a real eigenvalue spectrum. One important and physically motivated class of such Hamiltonians have what is called PT symmetry. The P operator stands for parity inversion, and the T operator stands for time reversal. A PT-symmetric Hamiltonian H commutes with the product of these operators: $[H, PT] = 0$. Not all PT-symmetric Hamiltonians have real eigenvalues, but often times they do – in this case, it is said that the PT symmetry is unbroken. If the eigenvalues have complex values, it is said that the PT symmetry is broken. Often times by tuning parameters in a problem, we can observe a PT transition between a broken phase and unbroken phase.

In this problem, we will consider a 2 state system with PT symmetry. Consider the Hamiltonian

$$H = \begin{pmatrix} \mathrm{i}a & b \\ b & -\mathrm{i}a \end{pmatrix}.$$

(a) Why is H not Hermitian?

(b) Explain why the time reversal operator T corresponds to complex conjugation.

(c) If the parity operator is defined to be

$$P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

show that $[H, PT] = 0$.

(d) Determine the time evolution of the system if $b = 0$. Comment on what happens.

(e) Determine the eigenvalues of H. Show that if $b < a$, the eigenvalues are complex, but if $b > a$, the eigenvalues are real. This transition is called the PT phase transition.

(f) Explain in words a simple physical argument for why the PT phase transition occurs.