Do Chemists Need Relativity?

Schrödinger ignored special relativity when developing quantum mechanics, and Dirac famously dismissed the possibility that relativity would be meaningful for quantum chemistry. Is this really true? In this problem, you will use dimensional analysis to determine whether or not relativity matters. Useful constants are: the speed of light $c \approx 3 \times 10^8$ m/s, Planck's constant $\hbar \approx 10^{-34}$ J $\cdot$ s, the mass of the electron $m \approx 10^{-30}$ kg, $e = 1.6 \times 10^{-19}$ C, $\epsilon_0 \approx 9 \times 10^{-12}$ F/m.

(a) Begin by estimating the length scale relevant for a nucleus with charge Ze (assume it is infinitely massive, for simplicity). Express your answer in terms of the fundamental parameters listed above, and Z, and then determine (up to Z) the numerical answer.[1]

(b) Estimate the "speed" v of an electron – if you like, the expected value of $|p|/m$ of the electron, to be more precise.

(c) Whenever $v \sim c$, relativistic effects become important. For what value of Z do relativistic effects matter? Are such nuclei present (anywhere) on Earth (and thus of interest to chemists)? Determine whether Schrödinger and Dirac were correct.

[1] Consider the classical Kepler problem for the electron.