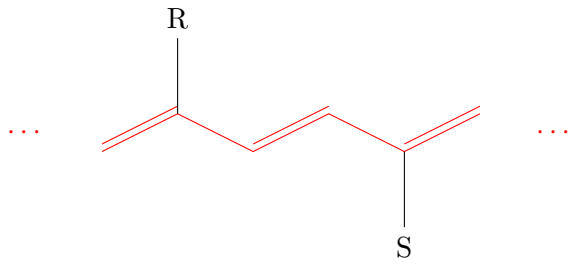
Pigment Molecules

A typical pigment molecule has the heuristic structure given below:



The bonds highlighted in red are special: they are called conjugated π bonds. Suppose there are N conjugated π bonds: then each bond contributes one "free" electron, of mass m, which may move up and down the chain of bonds freely. If each bond has length a, when N is large, we may thus approximate these electrons as moving in a particle in a box of width $L = a(N-1)$.

(a) Suppose N is an even number. Using the Pauli exclusion principle and the results for the particle in a box, describe which energy levels in the box are filled and which are empty.

(b) Now, suppose we send a photon of wavelength λ at the pigment molecule. What is the largest value of λ such that the photon can be absorbed by an electron in the pigment molecule? When the photon is absorbed, the electron must be able to jump to an unoccupied state in the box. You should find that

$$\lambda \approx KN$$

in the limit when $N \gg 1$ – what is the value of K?

(c) Evaluate numerically the value of K, given that $m \approx 9 \times 10^{-31}$ kg and $a \approx 10^{-10}$ nm. A typical pigment molecule might have a chain with $N \approx 20$. Does λ correspond to a photon in the visible spectrum?

(d) Suppose I give you 2 pigment molecules, one of which is red, and one of which is blue. Which pigment molecule do we expect has a longer chain of conjugated π bonds?