Evolution of a Star

Suppose that a star in space is made up of a very large number of He^4 nuclei of mass m_n, and a corresponding number of electrons N, with mass m_e to make the overall star charge neutral. The nuclei and electrons are evenly spread throughout the star, making electrical effects negligible.

Assume that the electrons act as nonrelativistic particles, and ignore interactions between electrons. The electrons are confined to move within the star.

(a) Neglecting the contributions due to electrons, calculate the gravitational potential energy of a star of radius r.

(b) Calculate the energy due to the degenerate electron gas, if the star has radius r, assuming that $T = 0$. For simplicity in this part, assume that the electrons live on a 3D torus (or cube, with periodic boundary conditions) with side length R.

(c) Calculate the radius of the star R, at which the potential is a minimum.

(d) Assume that $N = 10^{57}$. Calculate the Fermi energy E_F and the Fermi temperature T_F for the electrons. If the temperature of the star is approximately 10^7 K, is the assumption of $T \approx 0$ justified? You may use that the mass of an electron is about 9×10^{-31} kg, and the mass of a proton/neutron is about 1.7×10^{-27} kg.

(e) Compare E_F to $m_\mathrm{e}c^2$. Is the assumption of nonrelativistic electrons justified?

The answer to (e) should suggest that the non relativistic approximation may break down, so let's now assume that the electrons are ultrarelativistic.

(f) Neglecting the contributions due to electrons, calculate the gravitational potential energy of a star of radius r.

(g) Repeat (b), assuming that the electrons are ultrarelativistic.

(h) Show that there is only one star mass, $M = M_\mathrm{c}$, at which the star can have a minimum of potential energy at a finite non-zero radius. Evaluate this numerically, and compare it to the mass of the sun, $M_\mathrm{s} \approx 2 \times 10^{30}$ kg.

M_c is called the **Chandrasekhar limit.** Depending on the model used, the exact numerical answer can vary slightly, but it is always on the order of M_s.