Stoner Ferromagnetism

We often treat the electrons in a metal as a fermionic gas at $T = 0$ consisting of two types of electrons of mass m: spin up and spin down. Let n_+ be the number density of spin up electrons and n_- be the density of spin down ones. The electrons have two sources of energy: firstly, they have kinetic energy resulting from the Pauli exclusion principle; secondly, there is an interaction energy density given by

$$u_\mathrm{s} = \alpha n_+ n_-.$$

This is a result of spin-spin coupling between parallel spins. Assume that the number densities are given by

$$n_\pm = \frac{n \pm \delta}{2}$$

where $\delta \ll n$.

(a) Find an expression for the total energy density u, and expand it to fourth order in δ.

(b) The magnetization of the material is given by

$$M = m_0(n_+ - n_-) = m_0\delta.$$

Show that the energy density of the material is minimized when

$$M = \begin{cases} 0 & \alpha < \alpha_\mathrm{c} \\ \pm M_0(\alpha - \alpha_\mathrm{c})^\sigma & \alpha \geq \alpha_\mathrm{c} \end{cases}$$

and find expressions for α_c and M_0 in terms of the parameters of the problem, as well as the dimensionless parameter σ.

This phenomenon of spontaneous development of magnetization was developed by E.C. Stoner; we call this type of magnetization **ferromagnetism**.