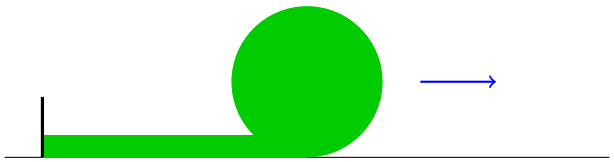
Rolling Hose

In this problem, we will consider the dynamics of a very long hose of length L and a mass M, uniformly distributed. At the beginning, this hose is rolled up into a thin disk of radius $R \ll L$, and you may treat it as a solid disk with a uniformly distributed mass.

Then, we hold down one end of the hose, and give it an initial push, so that the center of mass is traveling with speed v_0. As the hose travels, since we are holding down on one end, it will begin to unravel, such that after it has traveled a distance x, a length x of the hose has unraveled, as shown in the figure. Neglect frictional forces and gravity in this problem.



(a) Since we have argued that friction and gravity are irrelevant, where do the forces (if any) acting on the hose come from?

(b) Find the position $x(t)$ of the center of mass, as a function of time. You may assume that $x(0) = 0$, for simplicity.

(c) Sketch $x(t)$ carefully, in particular noting the following points: certainly, $x(t)$ cannot exceed L (the hose has fully unraveled). Does $x(t)$ reach this point in a finite time?

(d) Compute the linear momentum $p(x)$ of the hose as a function of the distance x is has traveled. Comment on the forces acting on the hose and check if your answer makes sense.