Sagnac Interferometer

A **Sagnac interferometer** is a classic device used to demonstrate the fact that light moves at different speeds in rotating coordinate frames. The set-up is as follows: monochromatic light enters a circular waveguide of radius r, which is rotating at a constant angular frequency ω, from one of two sources: each source sends the light into the waveguide traveling in the opposite direction. After traveling through the waveguide, which winds around n times, the light can exit. In particular, if the sources of light are in phase with each other, if the beams of light interfere after exiting the device, it is clear that it took a different time to travel around the waveguide in one direction than another.

(a) By considering cylindrical coordinates in a rotating frame, show that the metric in the laboratory frame is given by (using coordinates t, r, ϕ, z)

$$g_{\mu\nu} = \begin{pmatrix} -c^2 + \omega^2 r^2 & 0 & -\omega r^2 & 0 \\ 0 & 1 & 0 & 0 \\ -\omega r^2 & 0 & r^2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

(b) Show that the time it takes for a photon to travel once around the Sagnac interferometer is not the same for photons traveling in opposite directions, and that the difference in times is given by

$$\Delta t \approx \frac{4\pi\omega r^2 n}{c^2}.$$

assuming $\omega r \ll c$.

(c) Show that laboratory observers will see light travel at two different speeds depending on which direction it travels through the Sagnac interferometer. What are these speeds?