Dissociation of a Diatomic Gas

Let us consider the following simple model for a diatomic gas: a diatomic molecule consists of a pair of point-like atoms, each of mass m, separated by a rigid rod of length a. The rigid rod corresponds to the chemical bond, which requires an energy ϵ to break.

Now, at any finite temperature T, we know that *some* fraction of these diatomic molecules will actually break apart. Consider the reaction where one diatomic molecule breaks up into 2 monoatomic molecules. As usual, we can determine the equilibrium state of this system by comparing chemical potentials of the reactants. Let n_0 be the number density of diatomic atoms, at $T = 0$.

(a) Treating the diatomic molecule as a classical object, compute the chemical potential of both the diatomic molecules, and the monoatomic molecules.

(b) Determine the fraction α of diatomic molecules which will be disassociated in chemical equilibrium.

(c) For diatomic hydrogen, we have $m \approx 10^{-27}$ kg and $a \approx 10^{-10}$ m. At room temperature of $T \approx 300$ K, what is the fraction of hydrogen molecules which we expect to disassociate? Verify that this effect is negligible.