The Gas-Plasma Transition

In a volume V at temperature T, we have two types of particles: a number density n_+ of particles of charge $+q$ and mass m_+, and density n_- of particles of charge $-q$ and mass m_-. A pair of these particles can join together to create a neutral particle of mass m_0 and the neutral particles can disassociate into a positive and negative particle.[1] Some important examples of where this happens are in a plasma, where the particles are protons, electrons and hydrogen atoms, or in a semiconductor, electrons, "holes" and "excitons", effective particles caused by the interactions of electrons and holes.

For this entire problem, you may assume that in the absence of the possibility to create neutral particles, there is an equal number density n of positive and negative particles. Assume that it costs an energy ϵ to break up the neutral particle into the charged particles. Let d represent the number of spatial dimensions. Finally, you may assume that the gases of the 3 particles are all classical ideal gases, and use any results you'd like from the ideal gas theory we've worked out.

(a) Write down the condition of chemical equilibrium for the positive, negative and neutral particles.

(b) Find the number density n_0 of neutral particles, in terms of n, k_B, T, m_0, $m_\pm$, ϵ, and $\hbar$.

(c) What happens to n_0 at very low temperatures? Explain what is happening physically.

(d) What happens to n_0 at very high temperatures? How does n_0 scale with T?

(e) What is the temperature scale, T_0, at which we transition from a gas to a plasma? Note that this transition is *not* a phase transition, but a crossover. To simplify answers, you may work in the regimes where either ϵ is "very small" or "very big" (specify compared to what!).

It is believed that plasmas are in fact a distinct phase of matter from gases, although to find this we need to include interactions, something this model has not done yet.

[1] If the theory is an effective theory, we may not have $m_0 = m_+ + m_-$.