Double Stranded Polymers

In this problem, we will consider some simple models for polymer growth, with applications to biology. Let us begin by considering polymer growth with a single-stranded polymer. In biology, a single-stranded polymer typically grows as follows: an enzyme sits at the end of a polymer, and grabs a monomer from the surrounding cell and attaches it to the end of the current strand with rate αc_1, where α is some constant, and c_1 is the concentration of monomers. We will assume that a monomer falls off the end of the polymer, however, with rate β.[1]

Let c_n denote the concentration of polymers with n monomers attached (for $n \geq 1$). Also, define

$$K \equiv \frac{\beta}{\alpha},$$

which has the units of concentration.

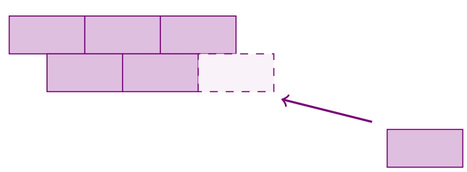
(a) Find an expression for c_n in terms of K and c_1, assuming a steady-state distribution. What constraints are there on c_1 and K for a steady-state distribution to exist?

(b) Find an expression for the concentration of polymers, c_{p}.

(c) Find an expression for the concentration of monomer building blocks, c_{m}.

(d) What is the average length $L = \langle n \rangle$ of a polymer?

(e) Suppose we now want to make $L \gg 1$. What does this imply about the fraction of polymers which are monomers?



Now, let us consider a double-stranded polymer, which grows as sketched above. We will assume that the rates are the same as above, except for the following reaction: when a polymer with $n = 2$ reduces to a pair of $n = 1$ monomers, that reaction occurs at rate γ, instead of β. In any realistic application, we would have $\gamma > \beta$, since there are fewer bonds connecting monomers together: notice how multiple monomers are all bonded together in the above diagram. Define

$$J \equiv \frac{\gamma}{\alpha}.$$

(f) Find an expression for c_n in terms of K, J and c_1, assuming a steady-state distribution.

(g) Find an expression for the concentration of polymers, c_{p}.

(h) Find an expression for the concentration of monomer building blocks, c_{m}.

(i) What is the average length $L = \langle n \rangle$ of a polymer?

[1]Thanks to Daniel Fisher for this problem.

(j) Show that it is possible to choose J and K such that it is possible to have $L \gg 1$, but a reasonable fraction of polymers be monomers.

(k) Can you think of a biological reason why using a double-stranded protein would be preferable to a single-stranded protein?[2]

[2] Suppose I want to build a large polymer quickly...