probability theory → random variables ★★★

Information Asymmetry in Investment

An entrepreneur has wealth W, which they want to invest in a business. However, the cost of setting up the business is $W + I$ (where $I > 0$), and thus they need to come up with money I by looking to an investor to loan them the money. The profit of the new business will be denoted by P, and because nobody knows the future, P is a random variable.

Assume that the going rate on investments is r: thus, anyone with money M who chooses to invest it in a safe investment can get money $M(1 + r)$ during the given time period. In this problem, you are to assume the investor will not see any profit margins on their investments: this is a reasonable assumption for a huge economy: if the investor required return $1 + r + \epsilon$, for some $\epsilon > 0$, then another investor would only ask for $1 + r + \epsilon'$, for $\epsilon' < \epsilon$. As this process continues, we see investors settling on simply $1 + r$. Assume for the purposes of this problem that the investor will always have enough money to invest.

(a) If there is information symmetry between the entrepreneur and the investor, then determine the payoff that the investor will receive, and the conditions under which the entrepreneur will take up the project.

Of course, in the real world, there is a very large information asymmetry between the investor and the business. A business has an incentive to lie to their investor and say there was no output: of course, legal action makes this less desirable, but that's part of the process to restore information symmetry! For the remainder of this problem though, we focus on ways in which the lack of information symmetry between firms and investors can reduce the efficiency of the economy.

Suppose that it costs the investor and the firm a constant cost c to verify the level of output. It turns out that, under some constraints, it can be proven using game theory that the optimal investment strategy is a so-called "debt contract", which goes as follows: there is some critical profit P'. if $P \geq P'$, then the firm will simply pay the investor P', and nobody will have to verify the profit. If $P < P'$, then the firm will pay the investor P, and verification will have to occur, so both the firm and investor will incur a cost c. You should be able to convince yourself that there is no reason for the firm to lie to the investor in this scheme.

We now solve a very simplified model under which an exact solution can be found. Consider the profit P to be a random variable distributed uniformly on $[0, 2\gamma]$. Our goal is to determine when, under information asymmetry, the investor will invest in the entrepreneur, and what P' will be.

(b) Show that the investor's expected profit on the deal, P_i, is given by

$$P_\text{i} = \begin{cases} P'\left(1 - \dfrac{P'}{2\gamma}\right) + \dfrac{P'}{2\gamma}\left(\dfrac{P'}{2} - c\right) & P' < 2\gamma \\ \gamma - c & P' > 2\gamma \end{cases}.$$

(c) Find P' in terms of I, r, γ and c.

(d) Under what conditions will the investor refuse to invest, for any P', in the entrepreneur?

(e) Show that the minimum expected profit required for the entrepreneur to be willing to start the business is higher with information asymmetry than with information symmetry, and determine by how much.

(f) Information asymmetry leads to many sub-optimal results for society. Sketch a plane with axes W and γ, and show the regions in which investment will take place (remember, this requires it to be the optimal choice for both the investor and the entrepreneur), both with and without information symmetry. Comment on the results.