Largest Random Variables

Consider a sequence of n iid random variables $X_1, \ldots, X_n$, drawn from the same probability distribution, which we assume is continuous. Denote $R_1 = 1$ and let R_j be the random variable for $j > 1$ defined by

$$R_j = \mathbb{I}(X_j > X_i \text{ for } j > i).$$

(a) Determine the probability distribution on R_j for $1 \leq j \leq n$.[1]

(b) Show that if $i \neq j$, R_i and R_j are independent.

Denote

$$S_n \equiv \sum_{j=1}^{n} R_j.$$

(c) Compute $\langle S_n \rangle$. Determine the dominant term when $n \gg 1$.

(d) Compute $\mathrm{Var}(S_n)$. Determine the dominant term when $n \gg 1$.

[1] For the next two parts, consider the problem if you switch the order of the random variables in the sequence. In particular, suppose you switch X_i and X_j for any $i \neq j$. What effect should this have?