probability theory → random variables ★★★

Management and Unions

Suppose a company has to incur a cost C in order to make and sell its product, and the burden of this cost must be split in some way between the managers and the workers (who are represented by a union – i.e., they must bargain collectively). If the company sells its product, we assume that the managers make profit M, and the workers make collective profit W.

We will assume that the negotiations over how to incur the cost proceed as follows, for simplicity. The managers are aware of the value of M. They pick a number $0 \leq \alpha \leq 1$ such that they would pay a cost αC, and the workers would pay a cost $(1-\alpha)C$. They then propose this value of α to the workers, who will in a deterministic manner, accept the deal or reject it. The managers profit is given by

$$\pi_{\text{manager}} = Z(M - \alpha C)$$

where

$$Z \equiv \Theta(W - (1-\alpha)C),$$

i.e., $Z = 0$ if the workers reject the deal, and $Z = 1$ if the workers accept the deal.

Now, this would be some nice deterministic problem except for that, under realistic circumstances, the manager does not actually know the value of W. For simplicity in this problem, we assume that W is a random variable drawn from a probability distribution with cumulative density $f(W)$, and that the manager knows this distribution, but not the value of W for the specific realization of the negotiation problem.

(a) Show that if the managers want to maximize $\langle \pi_{\text{manager}} \rangle$, they should choose α such that

$$1 - f((1-\alpha)C) = (M - \alpha C) f'((1-\alpha)C).$$

(b) For simplicity, assume that $W \sim \text{Uniform}(W_0, W_0 + \Delta)$. Given a W_0, Δ, C and M, what is the value of α that the manager will choose?

(c) Suppose that the manager's profit maximizing α is such that $\text{P}(Z = 0) > 0$. Is it possible that there is an α for which $\text{P}(Z = 0) = 0$ and both the union and the manager make a profit? Explain the economic and practical consequences of what you find.