Ambipolar Diffusion

In a plasma, ions of charge $+e$ and electrons of charge $-e$, can display interesting diffusive behavior in external electric fields. This effect is called **ambipolar diffusion** and is the study of this problem.

To begin, let us write down the diffusion equations for the ions (number density $n_\mathrm{i}(\mathbf{x},t)$) and electrons (number density n_e. If we place the plasma in an electric potential $\varphi(\mathbf{x},t)$, the diffusion equations are:

$$\frac{\partial n_\mathrm{i}}{\partial t} = D_\mathrm{i}\nabla\cdot\left[\nabla n_\mathrm{i} - \frac{en_0}{k_\mathrm{B}T_\mathrm{i}}\nabla\varphi\right],$$
$$\frac{\partial n_\mathrm{e}}{\partial t} = D_\mathrm{e}\nabla\cdot\left[\nabla n_\mathrm{e} + \frac{en_0}{k_\mathrm{B}T_\mathrm{e}}\nabla\varphi\right].$$

Here D and T represent the diffusion constants and temperatures for the ions/electrons. Since the ions are heavy and the electrons, we have $D_\mathrm{e} \gg D_\mathrm{i}$. Note that in a plasma the electrons and ions can actually effectively interact with *different* heat baths, and thus stay at different temperatures! In general, $T_\mathrm{e} \gg T_\mathrm{i}$, so you should assume that for this problem. In general, $n_\mathrm{i} \approx n_\mathrm{e} \approx n_0$, and so we have taken the liberty of already making this approximation in the diffusion equation, which will keep the equations linear and simpler to work with. This is not a big approximation, as you can argue for yourself, if you'd like. We finally combine these equations with Poisson's equation:

$$\nabla^2\varphi = -\frac{e}{\epsilon_0}(n_\mathrm{i} - n_\mathrm{e}).$$

(a) We typically define the Debye length scales for plasmas as

$$\lambda_\mathrm{e}^2 = \frac{\epsilon_0 k_\mathrm{B}T_\mathrm{e}}{e^2 n_0},$$
$$\lambda_\mathrm{i}^2 = \frac{\epsilon_0 k_\mathrm{B}T_\mathrm{i}}{e^2 n_0}.$$

These tell us about how far electric fields can propagate through a plasma without being screened. Suppose that we begin by slightly disturbing the ion/electron densities on a length scale much larger than the Debye length scales. Show that n_i and n_e will relax exponentially to each other in time. This is called the quasineutral approximation.

(b) If D_e is large, argue that the electrons will effectively reach "equilibrium" quickly. What constraint does this imply?

(c) Combine the approximations of the previous two parts and show that the ions have an effective diffusion constant of

$$D_\mathrm{i,eff} \approx D_\mathrm{i}\frac{T_\mathrm{e}}{T_\mathrm{i}}.$$

Comment on this result – why can the ions move around so fast in the presence of electrons?