## Size of a Bacterium

Suppose a simple bacterium is approximately a sphere of radius R. The bacterium requires oxygen to power the machinery within the cell which allows it to produce proteins and divide. Assume that the rate at which oxygen molecules are used per unit volume per unit time is a constant,  $\gamma$ , within the cell. Molecular oxygen, dissolved in water, has a diffusion coefficient D (assume this coefficient is the same both inside and outside the cell). Far away from the cell, the concentration of molecular oxygen is  $c_0$ .<sup>1</sup>

- (a) Solve the diffusion equation, assuming a time-independent solution, and use the answer to determine the largest possible value of  $\gamma$ ,  $\gamma_{\text{max}}$ , in terms of R, D and  $c_0$ .<sup>2</sup>
- (b) It is observed that γ ~ 20 mM/s in E. coli. bacteria, which are about 2 µm in diameter. The typical concentration of molecular oxygen in water is 0.2 mM, and the diffusion coefficient for oxygen is roughly 2 × 10<sup>-9</sup> m<sup>2</sup>/s. Do you expect that the growth rate of these bacteria is limited by the rate at which they can obtain oyxgen?

<sup>&</sup>lt;sup>1</sup>Thanks to Daniel Fisher for giving this problem.

 $<sup>^{2}</sup>$ Note that this upper bound is due to something un-physical which would happen in the solution to the diffusion equation.