Territoriality and Swarm Diffusion

Territorial and other spatial dynamics among populations, whether it be wolves, insect swarms, or unicellular organism colonies, is an important phenomena overlooked by simple random walks and diffusion. Let $n(x,t)$ be the number density of our population. We denote the conservation of number by

$$\frac{\partial n}{\partial t} + \nabla \cdot \mathbf{J} = 0$$

where $\mathbf{J}$ is the number flux of predators. We assume that

$$\mathbf{J} = \mathbf{J}_{\text{conv}} + \mathbf{J}_{\text{diff}}$$

where $\mathbf{J}_{\text{conv}}$ refers to the "convective" flux of creatures back to some sort of spawn point or den, and $\mathbf{J}_{\text{diff}}$ refers to the "diffusive" flux of the creatures, e.g. foraging for food. Empirical evidence shows that typically, our life forms have a decent sense of the geography near their den, which we place at $x = 0$, so a decent model for the convective flux is given by

$$\mathbf{J}_{\text{conv}} = -vn\frac{\mathbf{x}}{|\mathbf{x}|}$$

where v is the constant speed at which they move back to the den. We model the diffusive flux by

$$\mathbf{J}_{\text{diff}} = -\alpha n^c \nabla n$$

where $c \geq 0$ and α is a constant. $c = 0$ corresponds to pure diffusion, whereas $c > 0$ implies that the creatures are more likely to diffuse in packs, or when there are more of them together.

For simplicity, assume that the pack lives in a 1D geographic region: i.e., there is only 1 relevant spatial coordinate. Let us look for time-independent solutions of the diffusion equation.

(a) Show that for $c > 0$ the colony size has a *finite range*: i.e., there is an L for which $n = 0$ for $|x| > L$.

(b) In terms of the number of creatures, N, as well as α, c and v, find an expression for L.

(c) Why does this result break down when $c = 0$?