Uranium Critical Mass

Consider a sphere of uranium atoms of radius R. At a very small rate, these atoms will spontaneously emit high energy neutrons (mass $m \approx 2 \times 10^{-27}$ kg) with velocity v. After traveling a distance of $\lambda \approx 0.16$ m, a typical neutron will collide with a nucleus, be absorbed, and cause that nucleus to emit (on average) $\nu \approx 2.5$ neutrons of velocity v. In this problem, for simplicity assume that at any given time, half of the free neutrons are moving inwards towards the center of the sphere, and half are moving outwards.

(a) Suppose that n is the number density of free neutrons. Estimate the flux of free neutrons out of the edge of the sphere.

(b) Estimate the total rate of production of free neutrons in the sphere.

(c) There is a critical value of $R = R_c$ at which the sphere will "go critical" and cause a nuclear explosion, where the flux of free neutrons out of the sphere is balanced by the production of neutrons. Argue that this radius is[1]

$$R_c = \frac{3}{2}\frac{\lambda}{\nu - 1}.$$

Note that n has dropped out of the value of R_c. Also confirm that when $R > R_c$ a net number of neutrons is produced, and when $R < R_c$ most free neutrons leave the sphere before causing any other collisions. Plug in for numerical values; compare to the experimental value of about 10 cm.

(d) The mass density of uranium is about 2×10^4 kg/m^3. How massive would such a sphere of critical size be?

[1] The precise O(1) coefficient here is not particularly important. Just justify all of the approximations used.