

## Wall Street Does Not Random Walk

The foundations of mathematical predictions of the stock market are essentially based on the simple idea that stock prices roughly fluctuate like random walks. In this problem, we will comment on this assumption using intuition about random walks.

- (a) The Dow Jones typically fluctuates  $\sim 1\%$  in a day, and  $\sim 10\%$  in a year. Macroeconomic business cycles tend to be on time scales of 10 years. On time scales of a day, do you think stock price movements are dominated by deterministic motion or random, diffusive behavior?
- (b) The Dow Jones fell by more than 20% in a single day in 1987 (Black Monday). Working under the assumption that during a single day, the stock market is dominated by diffusive motion, estimate the probability that the stock market would change by more than 20% in a single day, and argue that a diffusive model does not properly capture the dynamics of stock prices.

Although our analysis was very simple, the basic conclusion that one needs a much more complicated model than the random walk to model stock prices is correct.

As an example of the danger that overly simplistic models have in mathematical finance, we can look to the market crash of 2008. One of the major drivers of this crisis was the fact that banks substantially underestimated the probability that many homeowners would simultaneously default, by more or less neglecting the possibility that a change in the macroeconomy could cause homeowners who could previously pay back their loans to suddenly default.