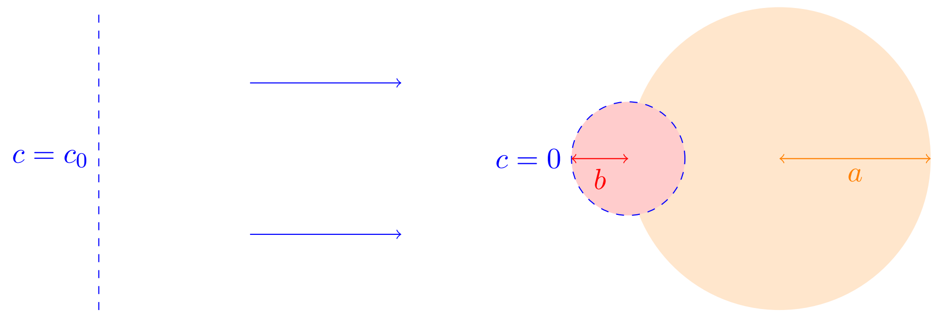
Bacterial Receptors

Bacteria determine the concentration of certain molecules, such as nutrients, by placing receptor proteins on the cell membrane. How many receptor proteins should a bacterium use? Let us model the bacterium as a sphere of radius a, and the receptor as a sphere of radius $b \ll a$. Suppose that the nutrients are at concentration c_0 very far from the cell, and they diffuse with a diffusion constant D. We consider time-independent solutions to the diffusion equation only. At the surface of each receptor, the concentration of the nutrients is 0 – i.e., the receptors absorb all of the nutrient molecules.



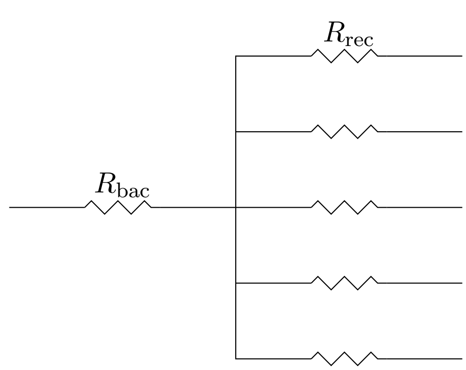
(a) Suppose that the entire surface of the bacterium is one giant receptors. What is the flux of nutrient molecules into the surface of the bacterium?

(b) Now suppose only a single receptor is present. Assuming that the nutrient can flow freely through the bacterium into the single receptor, what would be the nutrient flux?

(c) Compare the answers to parts (a) and (b). Argue that you do not need to cover the entire surface of the cell with receptors to get (up to some unknown constant factors) the same flux as in part (a). How many receptors do you need?

(d) It turns out that we can rigorously define an analogy between diffusion and electrical circuits. Justify that if we identify diffusive flux with electric current density and concentration with voltage, then the diffusion of nutrients from ∞ to a sphere of radius r is equivalent to the flow of current through a resistor. What is the diffusive resistance R of a sphere of radius r?

Now, suppose we put n receptors "uniformly" on the surface of the bacterium. To compute the flux, we now exploit our analogy to circuits, along with the following approximation: the nutrients first flow approximately to the surface of the cell, and then from there diffuse into one of the receptors. The analogous circuit is sketched below:



Here R_{bac} is the diffusive resistance of the bacterium, and R_{rec} is the diffusive resistance of a receptor, which you have computed in part (d).

(e) Use the circuit analogy to compute the flux into the n receptors. Sketch the flux J as a function of n, and comment on the results. Does your answer match up with your intuition from part (c)?