B-Form and S-Form DNA

Under the application of a large enough force, a strand of DNA can undergo a fairly sharp transition between what is called "B-form" DNA (normal) and "S-form" DNA (stretched). In this problem, we will consider a simple model which captures this effect.

Consider a strand of DNA, a polymer with N monomers (base pairs), which is fixed at one end, and pulled by a force of strength f at the other end. We assume that the monomers in DNA can move right or left a distance corresponding to their length. The monomers of B-form have a length of b, and the monomers of S-form have a length of s. For each monomer which is in S-form instead of B-form, we add an energy penalty of ϵ. The energy of the DNA strand under the applied force is therefore given by

$$E = \epsilon n_{\mathrm{S}} - f \sum_{i=1}^{N} x_i$$

where x_i is the displacement of a given monomer, and n_{S} is the number of monomers which are in S-form. Assume that this polymer is at inverse temperature $\beta = 1/k_{\mathrm{B}}T$.

(a) Using the form of the energy, explain why $\langle L \rangle = N \langle x \rangle$, where $\langle x \rangle$ is the expected displacement of a single monomer.

(b) Find an expression for $\langle x \rangle$.

(c) Sketch $\langle x \rangle$ vs. f if ϵ is small, and comment. Is there a sharp transition between two forms of DNA?

(d) Sketch $\langle x \rangle$ vs. f if ϵ is "large". Is there a sharp transition between two forms of DNA? What does it mean for ϵ to be large (i.e., in terms of other parameters, how large must ϵ be?)?