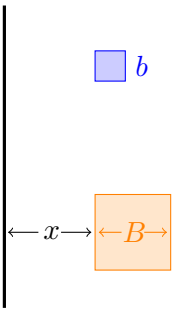
Pollen Grains

If a pollen grain is placed on the surface of a liquid, what happens? Due to the motions of the molecules in the liquid, it will move about randomly on the surface. But at some point, the grain will almost surely reach an edge of the water. It turns out that it tends to stay on the edge of the surface, due to the appearance of an entropic force. In this problem, treat the liquid as a 2-D gas made up of non-interacting particles at a fixed temperature T, consisting of squares of side length b. The pollen grain is a square of side length $B \gg b$. Ignore the kientic energy of the particles in this problem, as it will not affect the answer.



Let x be the distance from the nearest wall of the pollen grain. Assume that the surface of the liquid is a square of length L, with $L \gg B$, b, and that there are N total molecules of liquid. Let $N/L^2 = n$.

(a) Show that the free energy of the system is given by[1]

$$F \approx -Nk_{\mathrm{B}}T \log\left(\frac{1}{n} - \frac{b^2}{2} + \frac{B}{N}\min(b, x)\right).$$

(b) Assume $nb^2 \ll 1$. Show that the entropic force $F_{\mathrm{e}}(x)$ on the pollen grain is roughly

$$F_{\mathrm{e}}(x) \approx -nBk_{\mathrm{B}}T \cdot \Theta(b - x).$$

(c) Show that

$$\frac{\mathrm{P}(x \le b \mid \text{liquid molecules present})}{\mathrm{P}(x \le b \mid \text{no liquid molecules present})} \approx \frac{\mathrm{e}^z - 1}{z}$$

where z is a dimensionless parameter independent of T. The above expression assumes that both probabilities are much smaller than 1.

(d) Estimate the rough order of magnitude of z for a pollen grain on the surface of an ordinary liquid, and comment on the likelihood of finding the pollen grain along an edge. (The exact answer from part (c) may break down, but you will certainly get the point!)

[1] The particles are non-interacting, so the only contribution to F comes from entropy. The only entropy that matters is the entropy due to the number of available positions for the molecules in the liquid. Remember that liquid molecules are indistinguishable, and be careful with the geometry!