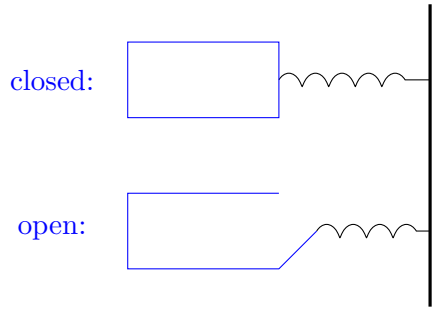
Stereocilia

Stereocilia are bundles of rod-like fibers in the inner ear that help you hear sounds. When experimenters went to measure the resistive force of stereocilia under displacements, they found a very strange result: the stereocilia behaved as if they had a negative spring constant for small displacements! In this problem, you will explore a simple model for why this happens.

We can model the stereocilia bundle as consisting of N rods, which are placed in parallel next to each other. Each rod can be thought of as a two-state system: a protein attached by a spring to the wall of the inner ear. In the "closed" state, if the displacement of the left end of the protein is x, then the resistive force of the spring is $-kx$. In the "open" state, however the protein unfolds a bit, and the resistive force becomes $-k(x+\delta)$, as we show in the picture below:



Suppose that the difference in free energies between the open and closed protein states is

$$F_{\text{open}} - F_{\text{closed}} \equiv \Delta.$$

(a) If the displacement of the left end of the protein is x, what is the probability that a given rod is closed? Remember, there is both the internal protein free energy and the energy required to compress the spring.

(b) What is the resistive force $F(x)$ of the bundle of N stereocilia? Assume that N is large enough that thermal fluctuations are negligible.

(c) Plot the function $F(x)$. Under what conditions will there be a regime with a negative spring constant?