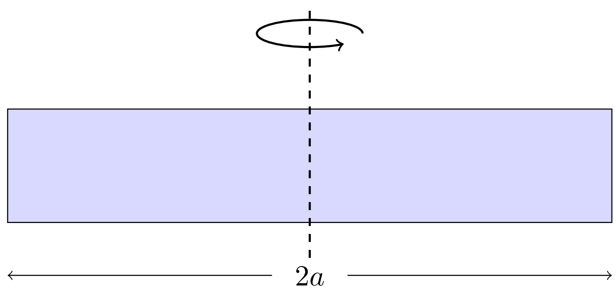
Uranium Centrifuges

The easiest type of nuclear weapon to build is a bomb based on uranium fission.[1] If a neutron strikes the nucleus of a ^{235}U atom, it can sometimes eject multiple neutrons and the uranium atom will decay into another isotope; if enough of these atoms are present (we say there is **critical mass**) then this reaction cascades downwards and a spectacular amount of energy is released. This is the concept behind a nuclear bomb.

Most uranium that is mined in nature does not have a high enough density of ^{235}U to be at critical mass, however; the predominant isotope is ^{238}U. The uranium must be **enriched**: i.e., have the ^{238}U removed, in order to put it in a pure enough state to be able to reach critical mass. The typical way of doing this is by gas centrifuges.[2] A crude model of a gas centrifuge is a long tube of length $2a$, rotating about a central axis with a constant angular velocity, such that the period of rotation is τ_0. Uranium does not appear in nature as a gas, but it is easy to attach the uranium atoms to flourine to make the gas uranium hexaflouride (UF_6).



Assume that the UF_6 gas consists of a pair of ideal gases, both at temperature T: one gas has the ^{235}U atoms (at number density n_{235}), and the other has ^{238}U atoms (at number density n_{238}). Let M and $M+m$ be the masses of these two species, respectively.

(a) Show that the number density of ^{235}U atoms, as a function of the distance x from the center of the tube, is given by

$$n_{235}(x) = n_{235}(0)\mathrm{e}^{-2\pi^2 Mx^2/k_\mathrm{B}T\tau^2}.$$

Infer a similar formula for the ^{238}U gas.

(b) Find an expression for α, defined as

$$\alpha \equiv \frac{n_{235}(0)}{n_{235}(a)}\frac{n_{238}(a)}{n_{238}(0)} - 1.$$

(c) Using $T = 300$ K, $m \approx 5\times 10^{-27}$ kg, $a = 1$ m, $\tau = 1$ s, give a rough estimate of α. You should find $\alpha \ll 1$.

[1] Of course I am not encouraging you to do this yourself (although as you should appreciate by the end of the problem, it is not easy). However, this is such a politically charged topic, and one that is fairly technical as well, that it is good for you to have a basic understanding of the physics behind crude uranium enrichment methods.

[2] The centrifuge concept is a very generic way of trying to separate things of different masses; it is also used to separate biological macromolecules, in a less violent application!

(d) Using the approximation $\alpha \ll 1$, simplify the expression for α.

In order to enrich the uranium, we remove gas from the ends of the centrifuge, where the heavier uranium atoms are more concentrated. A crude approximation is that once we put the gas into the centrifuge, we have to let it rotate q times in order to reach equilibrium. After that, we only keep the gas very close to $x \approx 0$, and throw the rest out. We then return the saved gas to the centrifuge and run it again, and repeat the process N times. Define

$$\eta \equiv \frac{n_{235}(0)}{n_{238}(0)}.$$

This corresponds to the "enrichment ratio" for the uranium.

(e) Suppose the gas initially had an enrichment ratio of η_0. After running the centrifuge N times, find an approximate expression for η, valid in the limit $\alpha \ll 1$, $N \gg 1$.

(f) Suppose we require an enrichment ratio of $R\eta_0$, for a constant $R > 1$. Neglecting the time it takes to remove the gas and start the centrifuges spinning,[3] calculate the time required to obtain the requisite enrichment ratio, in terms of R, a, m, τ, k_B, T and q.

(g) If the uranium is found naturally, the uranium will be about 0.7% ^{235}U. We need roughly 80% ^{235}U to make uranium enriched enough for a properly functioning nuclear weapon. Assuming $q \approx 100$, numerically evaluate the expression found earlier.

(h) Within reason, what would be the best parameter to change in order to reduce the time required for enrichment? Briefly explain.

[3] For this reason, centrifuges are typically arranged in very large arrays so that the enriched gas simply flows into the next centrifuge's chamber.