No 1D Gas Phase Transitions

Consider a gas of N particles of mass m, living in a 1D world with length L. Because the gas molecules are in 1D, so long as we assume that the gas molecules cannot "pass through" each other, since the ordering of the molecules on the line is independent of time, the particles are effectively *distinguishable*.

This is not the only interesting thing about the 1D world. Suppose that the Hamiltonian for these molecules is given by

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \sum_{i=1}^{N-1} U\left(x_{i+1} - x_i\right).$$

i.e., each molecule can only feel forces from adjacent molecules.

(a) Show that the partition function is given by

$$Z(T, N, L) = \left(\frac{mk_{\mathrm{B}}T}{2\pi\hbar^2}\right)^{N/2} \int_0^L \mathrm{d}y_1 \int_0^{L-y_1} \mathrm{d}y_2 \cdots \mathrm{e}^{-\beta(U(y_1)+U(y_2)+\cdots)}.$$

(b) The equivalent of pressure in 1D is a tension-like force τ. Show that the Gibbs partition function is given by

$$\mathcal{Z}(T, N, \tau) = \frac{T^2}{\tau^2}\left(\frac{mk_{\mathrm{B}}T}{2\pi\hbar^2}\right)^{N/2}\left(\int_0^\infty \mathrm{d}y\, \mathrm{e}^{-\beta(U(y)+\tau y)}\right)^{N-1}$$

(c) Show that this gas cannot go through a phase transition at any temperature, for any choice of $U(y)$.

The result of this problem breaks down if we allow for *long range interactions* where all gas molecules can interact with all others. It is also a sign of a much more generic phenomena about statistical and quantum field theories, in fact!