Black Holes

The entropy of a black hole is proportional to its area:

$$S = \frac{1}{4} C A$$

where C is a constant built up only out of k_B, G, c and $\hbar$ (with no constant coefficients).

(a) Use dimensional analysis to determine C.

(b) Assume that the black hole has mass m and is spherical, with radius r. Now, assume that a classical particle is moving at the speed of light, radially outward form the black hole. By using classical Newtonian gravity, determine the maximum radius r for which the particle can escape the presence of the black hole. This is the so-called Schwarzchild radius R. Find an expression for R in terms of m, G and c.

(c) Does entropy increase or decrease if two black holes collapse into one?[1]

(d) Use $E = mc^2$ to calculate the temperature T of a black hole. Express T as a function of m, or E.

(e) Assume a black hole emits radiation as a perfect blackbody:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\sigma A T^4.$$

Calculate the lifetime τ of a black hole. Assuming that it starts with $m = 10^{30}$ kg (order of magnitude of the mass of the Sun), of what order is τ?

[1] What property of a black hole do you think adds when the 2 black holes merge?