Earth's Ozone Layer

In this problem, we will show how the presence of the ozone layer could have a cooling effect on the Earth. For simplicity, let us consider the following set-up:

As in the picture, consider the Earth and ozone to be flat slabs emitting and absorbing radiation. The blue radiation corresponds to high energy radiation, which can either be emitted from the sun, or transmitted through the ozone, or reflected from the Earth's surface. The red radiation corresponds to radiation emitted from either the Earth or the ozone, which are at much lower temperatures than the Sun.[1] Let α be the albedo of the Earth (which is the same for both red and blue wavelengths), and suppose the Earth has perfect absorption. Let ϵ_1 be the emissivity of the ozone at red wavelength, and ϵ_2 at blue wavelength.

Denote with T_E the temperature of the Earth in the absence of the ozone layer, and T'_E the temperature of the Earth with the ozone layer.

(a) Show that

$$\left(\frac{T'_\mathrm{E}}{T_\mathrm{E}}\right)^4 = 1 + \frac{\epsilon_1 - \epsilon_2}{2 - \epsilon_1}\left(1 - (1-\alpha)\epsilon_2\right).$$

(b) It is known that $\epsilon_2 > \epsilon_1$ for the ozone layer. What do you conclude? Why might scientists be concerned about degrading the ozone layer?

[1] Neglecting the frequency dependence of the emitted radiation is a reasonable approximation here.