Energy from the Sun

In this problem, we will estimate the lifetime of the sun, and some other basic properties of nuclear fusion, based on simple data about our Sun and the Earth. Let's begin with estimating the mass and surface temperature of the Sun. You may find the following universal constants useful: $G \approx 7 \times 10^{-11}$ J · m/kg^2, and $\sigma \approx 6 \times 10^{-8}$ J/s · m^2 · K^4.

(a) Let's begin by estimating the mass of the Sun. Estimate that the orbit of the Earth around the Sun is a circle of radius 10^{11} m,[1] and that the period of revolution is 365 days. Using this data alone, determine the mass of the Sun.

(b) Estimate that the surface temperature of the Earth is 300 K, and that the Earth's radius is about 7×10^6 m. Assuming that the Earth is a perfect black body, what is the temperature at the surface of the Sun? You may also find useful that the radius of the Sun is about 10^9 m.

The Sun is a giant hydrogen fusion engine, in which hydrogen isotopes fuse into helium. Each time two hydrogens fuse into a helium atom, about 1% of the *rest energy* mc^2 of hydrogen is converted into heat. Use that $c = 3 \times 10^8$ m/s, and the mass of hydrogen is $m \approx 10^{-27}$ kg.

(c) How many reactions are occurring per unit time in the sun?

(d) Assuming that the hydrogen isotopes are spread uniformly throughout the interior of the sun, what is the density of hydrogen?

(e) Since two hydrogens have to come together to fuse into helium, we can estimate that the rate of reactions per unit volume should be

$$r = \beta n^2$$

for some constant β, if n is the number of hydrogens per unit volume. Estimate the constant β.

(f) Assuming that at time $t = 0$, all of the mass of the sun consists of hydrogen, estimate the *lifetime* of the sun. Assume that the reaction rate stays constant for the entire lifetime, for simplicity. Scientists believe that the "lifetime" of the sun[2] is 10^{10} years – compare your result to this answer.

(g) Actually, most of the fusion reactions occur only in the inner 1% of the volume. Which of the estimates you made above has to change?

[1] This can be estimated by considering the transit of Venus – read more online!.

[2] Actually, when this lifetime expires, the star will begin to turn into a red giant star. So this argument is a bit crude.