Freeze Time of a Magma Ocean

It is generally believed that the Moon was formed by a large asteroid hitting the Earth and sending a very large chunk of matter into orbit. After this collision, it is reasonable to expect that the Earth's surface would be a magma ocean. How long would it take for the surface of this magma ocean to freeze?

It is reasonable to assume that only the upper layer of the Earth's magma ocean would freeze. This is because even today, parts of the interior of the Earth are liquid magma. Therefore, it is unlikely that the entire Earth would freeze. In this problem, we will approximate that the thin layer of magma at the surface of the Earth does not "interact" with the magma beneath the surface, and therefore assume that only the thin layer of magma forming the future crust will cool down.

In this problem, assume that the temperature of this magma ocean is about $T = 2000$ K. The magma has a mass density of $\rho \approx 5000\ \mathrm{kg/m^3}$, and has a latent heat of freezing of about $L \approx 3 \times 10^5$ J/kg. The Earth's radius is approximately $R \approx 7 \times 10^6$ m, and the thickness of the crust is typically $\delta \approx 10^4$ m. Recall that the Stefan-Boltzmann constant is $\sigma \approx 6 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}$.

(a) Show that the time required for the crust to freeze is approximately

$$t = \frac{L\rho\delta}{\sigma T^4}.$$

Why does it not depend on the Earth's radius?

(b) Plug in for the numerical values given. Express the answer in years, using that a year is approximately 10^7 s.

(c) We have neglected the energy being added to the Earth by the light incident from the Sun. The intensity of light from the Sun is approximately $I_{\text{Sun}} \approx 500\ \mathrm{W/m^2}$. Is this effect negligible?

(d) Do you think a more realistic calculation would increase or decrease t, as found earlier? Explain your answer.