Snowball Earth

In class, we discussed estimating the temperature of the Earth by assuming the Earth was heated solely by incoming solar radiation. A very interesting phenomenon emerges when one assumes that the Earth will reflect some of the energy back to space at temperature dependent rates. We define a quantity called the **albedo** α, such that the power absorbed by the Earth is given by

$$P_{\text{absorbed}} = (1 - \alpha(T))P_{\text{incident}},$$

and assume that α is T dependent. Assume that the Earth radiates as a perfect blackbody.

(a) Find an equation which T must satisfy if the planet is in equilibrium, using energy balance. Express your answer in terms of I, the intensity of incident radiation, α, T and any other important constants.

(b) Suppose for simplicity that

$$\alpha(T) = \begin{cases} \alpha_1 & T < T_1 \\ \alpha_1 + (\alpha_2 - \alpha_1)\dfrac{T - T_1}{T_2 - T_1} & T_1 \leq T \leq T_2 \\ \alpha_2 & T > T_2 \end{cases}$$

with $\alpha_1 > \alpha_2$. Discuss the various possible equilibria of the system. Depending on the values of α_1, α_2, T_1 and T_2 you should find there can be either 1, 2 or 3 equilibria. Discuss the stability of the equilibria as well.

(c) Suppose that I is varying. Show that if

$$\frac{T_1^4}{T_2^4} > \frac{1 - \alpha_1}{1 - \alpha_2}$$

then there are values of I with 3 possible T. Sketch the various equilibrium T vs. I, making note of which equilibria are stable and which are unstable. Discuss the implications of the planetary "phase transition" predicted.

(d) Could an early Earth have undergone a sudden phase transition to "snowball Earth"? Crude guesses at the parameters above are $\alpha_1 = 0.5$ and $\alpha_2 = 0.25$, with $T_2 = 280$ K and $T_1 = 265$ K. This results from the fact that ice is much more reflective than sea water. Using the estimate $T = 290$ K for the modern Earth's temperature, show that there is a possible "snowball Earth" phase for the same amount of (effective) incoming radiation, and determine the temperature of this snowball.