# Temperature of the Human Body

The human body is quite adept at regulating its own temperature. In this problem we will explore this regulation with some simple physical models.

(a) Suppose that the body is absorbing heat at rate $P$: i.e., $P = \mathrm{d}Q/\mathrm{d}t$. We also know that

$$P = C\frac{\mathrm{d}T_\mathrm{b}}{\mathrm{d}t}$$

where $T_\mathrm{b}$ is the temperature of the body. Estimate the heat capacity $C$ of the body.[1] Assume that the person weighs 60 kg.

(b) The body loses heat due to the fact that it is radiating as a black body at temperature $T_\mathrm{b}$. The outside air is at temperature $T_\mathrm{a}$. Assuming that $T_\mathrm{a} \approx T_\mathrm{b}$, show that the rate of heat loss to the environment due to radiation, $P_\mathrm{rad}$, is given by

$$P \approx -\lambda(T_\mathrm{b} - T_\mathrm{a})$$

and find an expression for $\lambda$ in terms of relevant parameters, and any necessary fundamental constants. Estimate that the surface area of a person is about 1 $\mathrm{m}^2$ when finding $\lambda$.

(c) Now, suppose that $P = P_\mathrm{met} + P_\mathrm{rad}$, where $P_\mathrm{met}$ is a constant. To figure out this constant, use the fact that $T_\mathrm{a} \approx 4.0 \times 10^{-21}$ J, and the "optimal" body temperature $T_\mathrm{b}^* \approx 4.1 \times 10^{-21}$ J . If $T_\mathrm{b}^*$ does not change in time, determine $P_\mathrm{met}$. You are supposed to eat about $8 \times 10^6$ J (2000 kcal) worth of food a day – compare your answer to this number.

(d) What is the time scale of the dynamics of $T_\mathrm{b}$?

(e) Now, if the body temperature strays too far from $T_\mathrm{b}^*$, the body begins to activate control mechanisms to push the temperature back. If the body gets too hot, it can begin to release energy by sweating. Sweating occurs by allowing water to evaporate on the surface of the skin. For each water molecule that evaporates, about $10^{-19}$ J of energy is released. If the body can release about $\alpha \sim 3$ g/s of water (sweat) per second on to the surface of the skin, estimate the amount of energy which can be released by sweating, every second. Assume that all of this water instantly evaporates.

(f) If the body gets too cold, it can begin to shiver to release energy. The shivering of muscles generates heat, roughly by causing the dephosphorylation of ATP. Assuming each dephosphorylation of ATP releases $10^{-19}$ J, and assuming that shivering can release the same amount of heat as sweating removes, determine the number of ATP per second which must be consumed during shivering.

(g) The body cannot instantly turn on the mechanisms described in parts (e) and (f). Let $R$ be the maximal (absolute value) of power generated by these mechanisms. Instead, when $T_\mathrm{b} - T_\mathrm{b}^*$ is small, the power released by the body can be approximated by

$$P_\mathrm{control} \approx R\frac{T_\mathrm{b} - T_\mathrm{b}^*}{\delta}$$

where $\delta \approx 3 \times 10^{-23}$ J. Argue that the control mechanisms (sweating and shivering) decrease the time scale over which the temperature will relax to $T_\mathrm{b}^*$ substantially, and find the new time scale.

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[1] What material do you think is the dominant contributor to the heat capacity? You'll need to look up relevant facts about basic materials relevant in biology, if you don't know them already.