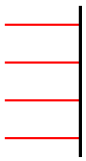
Bamboo

Suppose we have a collection of N bendable elastic rods with stiffness constant H and length L. Each rod buckles when a critical force

$$F_c = \pi^2 \frac{H}{L^2}$$

is applied to it. How should we arrange these rods to increase the critical force to the maximal force F_c^{tot} possible?[1]

(a) Let's begin by considering the following simple set up, where the rods are simply placed in parallel, but with each allowed to move on its own:

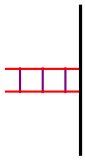


What is the critical force F_c^{tot} for the combined rods?

(b) Now, suppose that we bunch all of the rods up together. This time, the rods are packed tight enough together that they are forced to move together – i.e., they will only all buckle simultaneously:

What is the critical force F_c^{tot} for the combined rods?

Now, let us consider looking to nature. The structure of bamboo is sketched below:



The rods are arranged in a ring, only a single rod thick. Then, we place additional rings over the rods, which simply bind the rods together, at some fixed spacing.

(c) Begin by assuming that the configuration will move as one unit – i.e., the individual rods will not buckle. What is F_c^{tot} in this configuration?

(d) How many of the support rings are needed to ensure that the individual components do not buckle?

[1] Thanks to Daniel Fisher for giving this problem.