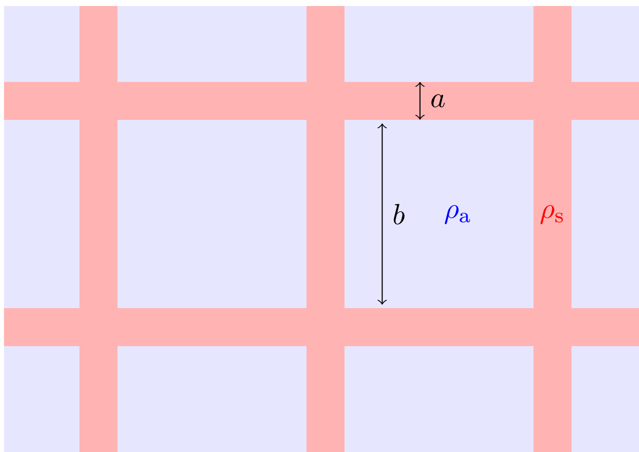
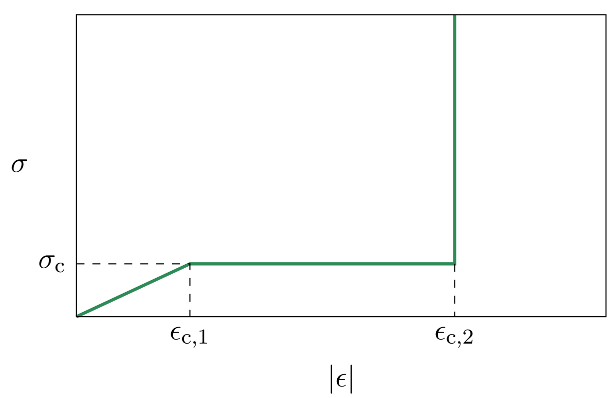
Cellular Solids

In this problem, we will explore the mechanical behavior of solids which are made up of a cell-like structure, consisting of an array of hard material suspended in a very light material. Many materials with this rough structure also occur as solids: for example, the cellular structure of plants consists of an array of tough cell walls filled with fluid. Below we see simple example of a solid with a cell-like structure. The blue material is, e.g., a fluid such as air, with density ρ_a. The red material is hard solid structure with density ρ_s.



For simplicity, we assume a 3 dimensional solid with the structure depicted in the figure above. An array of hard solid rectangular rods of density ρ_s and thickness a is placed in a cubic lattice structure, with a spacing b in between rods. Assume that $b \gg a$. Inside of the resulting array we place a material with density ρ_a; assume $\rho_\mathrm{a} \ll \rho_\mathrm{s}$.

(a) What is the resulting mass density, ρ, of the cellular structure?



Now let's turn to the mechanical behavior of such a solid under compression. As shown in the figure, there are essentially three different types of behavior. Let us describe why this occurs, and estimate σ_c. Assume that E_a and E_s are the Young's moduli of the filler and solid material, respectively.

(b) Let's begin by looking at the elastic regime. Argue that the effective Young's modulus of the cellular solid is given by

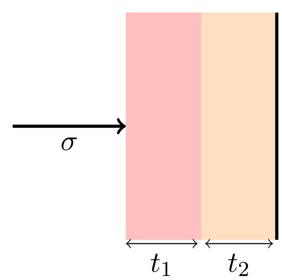
$$E \approx E_\mathrm{a} + \left(\frac{a}{b}\right)^2 E_\mathrm{s}.$$

When σ reaches σ_c, something dramatic happens. In a normal solid, when it reaches the yield stress it will begin to exhibit plastic behavior. Here we also see that when the solid yields, it exhibits nonlinear deformations: just like with plastic behavior, these deformations are irreversible. However, the reason they occur is dramatically different. The yielding of the cellular solid has nothing to do with plasticity: instead, what happens is that the stresses become too severe for the microscopic cell structure to withstand them, and so the structure of individual cells begins to fail.

(c) Estimate σ_c in terms of a, b, E_a and E_s.[1]

(d) Finally when $\epsilon = -\epsilon_{\mathrm{c},2}$, the cellular solid essentially no longer compresses. This transition is often as dramatic as the first. It is a reasonable assumption that $\epsilon_{\mathrm{c},2} \gg \epsilon_{\mathrm{c},1}$. Suppose someone tells you that after empirical testing, they found that $\epsilon_{\mathrm{c},2} \sim (b/a)^\nu$. What do you think is the value of ν? Explain your answer by describing the mechanism which leads to this second transition in behavior.

Materials with this sort of cellular structure are becoming quite popular in modern engineering applications. The main reason for this is that the deformation of these solids absorbs a lot of energy: this makes them valuable as protective materials in packaging, or in vehicle bumpers, etc. Let us try and understand why these materials are good at serving this role.



Let us imagine the following situation. A uniform compressive stress σ pushes on a slab of material 1 of thickness t_1, which pushes on a slab of material 2 of thickness t_2, which itself pushes on a hard surface. t_1 and t_2 are measured before compression. Material 1 has Young's modulus E_1, and represents something that we would like to not absorb much energy. Assume the slabs have area A.

(e) Suppose that material 2 is an elastic solid with Young's modulus E_s. After the system has reached equilibrium, what fraction of the energy has been absorbed by material 1? Comment on the result with respect to the application of designing a packaging material.

(f) Now suppose material 2 is made out of the cellular solid. If σ is just barely larger than σ_c, what fraction of the energy has been absorbed by material 1? Comment on the result with respect to the application.

[1] You can use purely elastic theory to estimate this. Which part of the structure do you think will fail?