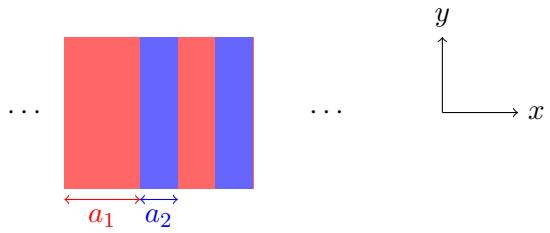
Composite Materials

Many modern materials are **composites** made up of multiple different types of materials. In this problem, we explore a simple composite material made up of two materials, labeled 1 and 2. Material 1 has Young's modulus E_1, and material 2 has Young's modulus E_2. As shown in the figure, we make the material out of planar sheets of these materials with thicknesses a_1 and a_2 respectively, laid flat on top of each other. Let's see what this does to the elastic properties of the material.



(a) First compute the Young's modulus in the parallel direction, $E_\parallel$: i.e., $\sigma_{yy} = E_\parallel \epsilon_{yy}$. Show that

$$E_\parallel = \frac{a_1 E_1 + a_2 E_2}{a_1 + a_2}.$$

(b) Now compute the Young's modulus in the perpendicular direction, $E_\perp$: i.e., $\sigma_{xx} = E_\perp \epsilon_{xx}$. Show that

$$\frac{a_1 + a_2}{E_\perp} = \frac{a_1}{E_1} + \frac{a_2}{E_2}.$$

(c) Compare $E_\parallel$ and $E_\perp$: which is larger?

Depending on the material properties we want, we should thus be careful about how we layer the two materials and how we arrange them in the material.