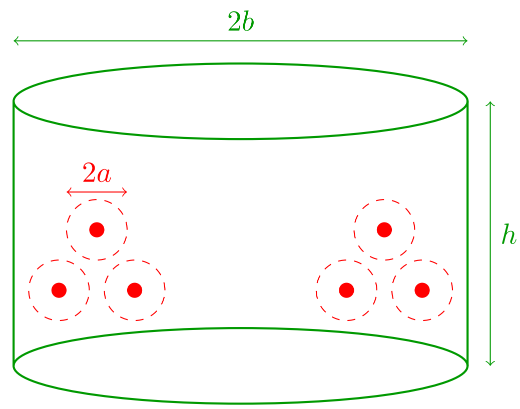
DNA Packing in Virus Capsids

A virus transmits an infection to an organism by carrying strands of foreign DNA in a **capsid**, and then using the capsid to inject the foreign DNA into the host cell, where the DNA can then be replicated as that cell divides. Since the foreign DNA causes the production of new (harmful) proteins, this causes the malignant infection. In this problem, we will get a crude sense for the energies required to pack DNA into a capsid.

Let us approximate DNA as a thin elastic cylindrical rod of Young's modulus Y and radius ρ. For the purposes of packing, we assume that the DNA packs as if it was a hard elastic rod of radius a, but has the same energy of packing as it would given its actual size. Treat the capsid as a cylinder of radius b and height h. We take the DNA to have net length L, and assume $L \gg b, h \gg a > \rho$.



We want to understand the energy required to pack DNA into this capsid. In general, there will be two contributions: one from elastic energy of DNA bending, and one from the interaction between DNA strands, which are typically charged. We can write this as

$$E = E_\mathrm{e} + E_\mathrm{i}$$

with E_e coming from elastic contributions, and E_i from electrostatic interactions between the strands. We determine E_i experimentally as[1]

$$E_\mathrm{i} = \frac{u_0}{a_0}\mathrm{e}^{-a/a_0}L,$$

with a_0 a length scale and u_0 an energy scale, so that the units work out. In this problem you may assume that $L, b, h \gg a_0 \gg \rho$.

We first begin by computing E_e, the elastic energy required to pack the DNA into the capsid.

(a) Find the elastic energy E_hoop of a single, circular loop of DNA of length $L \gg \rho$.

(b) In biology, the dominant energy scale is $k_\mathrm{B}T \approx 4 \times 10^{-21}$ J, at room temperatures. We can thus express

$$E_\mathrm{ring}(L) = \pi k_\mathrm{B}T\left(\frac{\xi}{L}\right)^\alpha$$

where α is the appropriate power you found above, and ξ is a length scale called the persistence length. Find an expression for ξ in terms of relevant parameters.

[1] The form is not too unreasonable, starting from the theory of electrostatics in salty solutions, either.

(c) In a typical cell, dephosphorylating an ATP releases about $20k_\mathrm{B}T$, and $\xi \approx 50$ nm for DNA. How many ATP would we need to release to wind DNA once around a viral capsid, using the value of b above?

(d) Find the Young's modulus Y of DNA, given that $\rho \approx 2$ nm.

(e) Now, suppose we pack the DNA into the capsid by assuming that the DNA acts effectively as a hard cylinder of radius a. Since $L \gg h, b \gg a$, we can approximately write

$$E_\mathrm{e} = \int_r^b \pi k_\mathrm{B} T \xi \frac{n(R)}{R} \mathrm{d}R$$

where r is the minimum radius required to pack DNA of length L in the capsid and $n(R)$ is the approximate number of loops per unit radius R that we can pack. By finding $n(R)$, $r(L)$, and performing the integral, show that

$$E_\mathrm{e} = -\frac{\pi}{\sqrt{3}} \frac{\xi h}{a^2} k_\mathrm{B} T \log\left(1 - \frac{\sqrt{3}}{2\pi} \frac{a^2 L}{b^2 h}\right).$$

(f) Using physical considerations, find L_max, the maximum possible length which will fit in the capsid.

Now that you have expressions for E_e and E_i in terms of a and L, you can determine $E(L)$ by minimizing the sum with respect to a, for a given L.

(g) By considering what happens for $L \ll L_\mathrm{max}$ and for $L \lesssim L_\mathrm{max}$, sketch the curve of E vs. L. Verify that your guess was correct by showing a numerical plot.

(h) How much energy would it cost to pack $L = 0.01L_\mathrm{max}$? What about $0.1L_\mathrm{max}$? What about $0.9L_\mathrm{max}$? Find your answers numerically, and express them in terms of the approximate number of ATP the host cell must expend to push the DNA in.