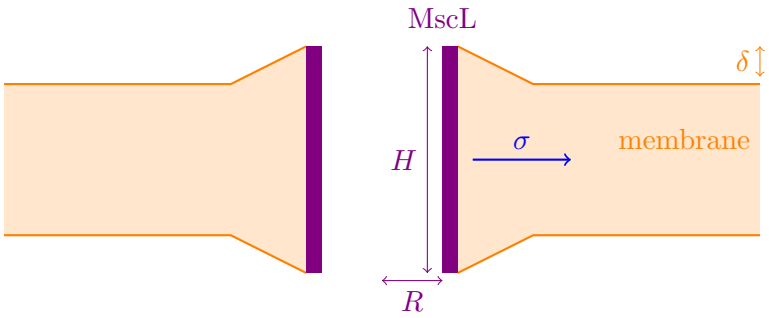
MscL

MscL is a channel gate protein in the phospholipid membrane of a biological cell. When the protein is pulled on by stresses in the membrane, a hole opens up in the center and allows for transport in and out of the cell. In this problem, we will determine a very simple model for determining how much stress is required to open up the protein.

Let us consider the MscL to be a flexible cylinder of a fixed height H and a variable radius for the hole, R, which is bounded above: $R < R_+$. There are two contributions to the energy of the MscL: $E = E_\sigma + E_\mathrm{m}$. The first, E_σ, comes from the applied stress σ that we apply to MscL to try and open it. The second contribution, E_m, comes from the fact that the lipid membrane must bend.



(a) In terms of H, R, and σ, estimate E_σ – the energy change in the system if the MscL has been expanded to radius R.

To estimate the membrane contribution to the energy, we will use the fact that we can approximate the lipid membrane as an elastic sheet with vertical deformation height $u(\mathbf{x})$ with a hydrophobic interaction:

$$E_\mathrm{m} = E_\mathrm{m,el} + E_\mathrm{m,hydro} = \frac{K}{2}\int \mathrm{d}^2\mathbf{x}\left[\nabla^2 u\right]^2 + \frac{J}{2}\int \mathrm{d}^2\mathbf{x}\, u^2.$$

Here K and J are positive constants.

(b) What are the equations of motion for the deformation of the membrane?

(c) Approximate that R is large enough so that the deformation of the membrane can be locally approximated as a 1D problem: $u(x, y) = u(x)$. The appropriate boundary conditions are that $u(\infty) = 0$, $u(R) = \delta$, and $u'(R) = 0$. Determine the solution to the equation of motion and the subsequent value of E_m.

(d) What is the critical value of the stress, σ_c, for which the MscL's preferred energetic state is the open state?