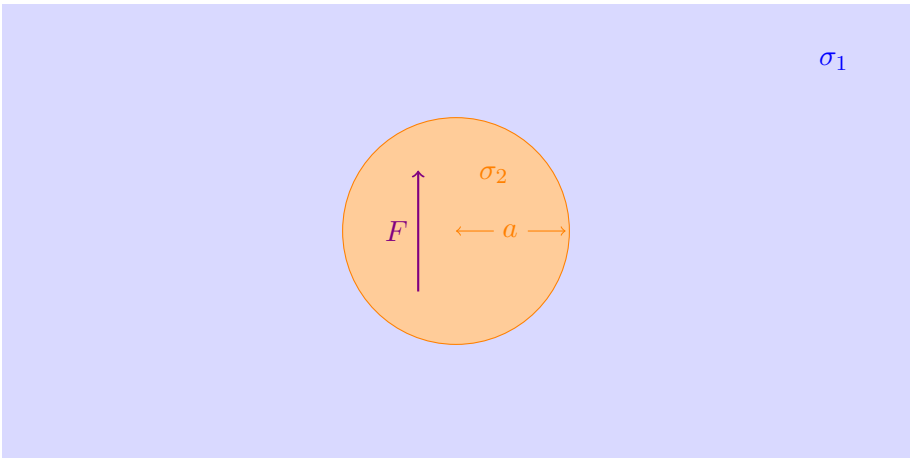
# Spherical Battery

A sphere of radius $a$ and constant conductivity $\sigma_2$ is placed in a uniform medium with conductivity $\sigma_1$. A chemical force $\mathbf{F}$ pushes charge carriers (electrons) upwards in the sphere as shown in the diagram below, such that Ohm's law inside of the sphere becomes $\mathbf{J} = \sigma_2(\mathbf{E} + \mathbf{F})$. Assume that $\mathbf{F} = F\hat{\mathbf{z}}$.




(a) By matching boundary conditions on the surface of the sphere, find the electric fields and currents everywhere in space, in terms of $a$, $F$, $\sigma_1$, $\sigma_2$ and $\epsilon_0$.

(b) What is the current $I$ flowing out of the top half of the sphere?

This can be used to crudely model a battery as follows. Let $P_1$ be the power dissipated outside of the sphere, and $P_2$ be the power dissipated inside of the sphere.

(c) Compute $P_1$; then use the fact that $P_1 = IV_1 = I^2R_1$ to find expressions for $V_1$ and $R_1$, the effective external voltage/resistance of the battery.

(d) Compute $P_2 = IV_2 = I^2R_2$ as well. Note that the power density inside the sphere is given by $\mathbf{J} \cdot (\mathbf{E} + \mathbf{F})$.

(e) Find expressions for the total resistance $R = R_1 + R_2$ and the total voltage $V = V_1 + V_2$. Show that $V = 4aF/3$.