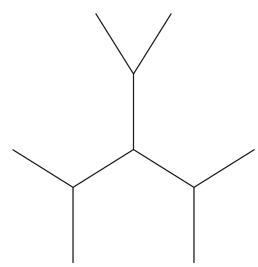
Electron on a Tree

Consider the following Hamiltonian for an electron which is allowed to move on a tree:

$$H = -\mu \sum_i c_i^\dagger c_i - \eta \sum_{i,\delta} c_i^\dagger c_{i+\delta}$$

where c_i is a fermionic annihilation operator on site i, and $i+\delta$ refers to a neighbor site of i. A tree is sketched below, for the example where there are 3 neighbors to each site:



In this problem, we will consider the general case where there are $m \geq 2$ neighbors for each site.

(a) Show that the exact Green's function in the frequency domain for the particle to remain in one place is given by

$$G(0,\omega) = \frac{\mathrm{i}}{\mathrm{i}\omega + \mu - \dfrac{2m\eta^2}{\mathrm{i}\omega + \mu + \sqrt{(\mathrm{i}\omega+\mu)^2 - 4(m-1)\eta^2}}}.$$

To do this, you need to be a little bit careful. For the *first* jump off of the node of interest, there are m possible locations to jump to. For the remaining jumps however, there are only $m-1$ options (since going back does not count when computing the self-energy – why?).

(b) Show that the density of states per site is given by

$$n(E) = \frac{m\sqrt{4(m-1)\eta^2 - E^2}}{2\left(m^2\eta^2 - E^2\right)} \Theta\left(4(m-1)\eta^2 - E^2\right).$$

(c) Sketch $n(E)$ for $m = 2$, 3 and 6, and comment on important differences.