Quantum Quench Dynamics

In this problem, you will explore the dynamics of a quantum *quench*, where the parameters of the quantum Hamiltonian and/or action are suddenly changed. In general, this has very interesting effects on the quantum dynamics of a system: you will look at a very simple case in this problem.

We begin by considering the harmonic oscillator given by Hamiltonian

$$H = \frac{p^2}{2} + \frac{\omega^2 x^2}{2}.$$

Do not yet assume there is a quench: this is the simple harmonic oscillator you are familiar with.

(a) Show that the following is an *operator relation* for the harmonic oscillator:[1]

$$x(t) = x(0)\cos(\omega t) + \frac{p(0)}{\omega}\sin(\omega t).$$

Now, let us perform a quantum quench on this system, by replacing $\omega \to \omega(t)$ with

$$\omega(t) = \begin{cases} \omega_0 & t < 0 \\ \omega & t \geq 0 \end{cases}.$$

(b) Let $|0\rangle$ correspond to the vacuum state of the harmonic oscillator for $t < 0$. Show that if $t_2 \geq t_1 > 0$, you can use the result you found above to find with a bit of algebra that:

$$\langle 0|x(t_2)x(t_1)|0\rangle = \frac{\mathrm{e}^{-\mathrm{i}\omega(t_2-t_1)}}{2\omega} + \frac{\left(\omega^2 - \omega_0^2\right)\cos(\omega(t_2+t_1)) + (\omega-\omega_0)^2\cos(\omega(t_2-t_1))}{4\omega^2\omega_0}.$$

Comment on the result.

Now, let's consider the following real scalar field theory:

$$\mathcal{L} = \frac{1}{2}(\partial_t\phi)^2 - \frac{1}{2}(\partial_i\phi)^2 - \frac{1}{2}m(t)^2\phi^2$$

with the time dependent mass term

$$m(t) = \begin{cases} m & t < 0 \\ 0 & t \geq 0 \end{cases}.$$

This corresponds to a scalar field where we suddenly quench the mass and turn it off. Just as in the case of a single oscillator, we expect that the switch of what we mean by the vacuum will cause a spontaneous excitation of fields. However, can we see nontrivial correlation functions of these fields arise after the quench? The answer is almost certainly yes, and you will now show a simple example of this.

(c) Working in 3 spatial dimensions, evaluate the correlator $\langle 0|\phi(r,t)\phi(0,t)|0\rangle$, where the ϕ fields are evaluated in position space. The two fields are at the same time $t > 0$, but separated by a spatial distance r. To do this, expand $\phi(x,t)$ in terms of creation/annihilation operators for each Fourier mode. Show that in the limit that $mt \gg 1$, the correlator becomes approximately

$$\langle 0|\phi(r,t)\phi(0,t)|0\rangle = \begin{cases} \dfrac{m}{16\pi r} & 2r < t \\ 0 & 2r > t \end{cases}.$$

(d) Give a physical interpretation of why there is a qualitative change at $t = 2r$.

[1] To do this, determine $a^\dagger(t) = U(t)a^\dagger U^\dagger(t)$ and $a(t)$ defined similarly. To do that, I would consider evolving the state $a^\dagger|\psi\rangle$ (for any $|\psi\rangle$) in time.