probability theory \rightarrow stochastic processes

Cell Growth

Let us consider a simple model for cell growth. We start with a single cell, e.g. a bacterium, at time t = 0. That cell will split into 2 cells at a constant rate α . Then, each of those 2 cells will split into 2 cells with rate α , etc. You can think of this as the continuous time version of the birth-death process, e.g.

(a) Let $p_n(t)$ be the probability that there are n cells at time t. Show that

$$p_n(t) = e^{-\alpha t} (1 - e^{-\alpha t})^{n-1}.$$

The master equation is probably the easiest way to do this.

(b) Find an expression for $\langle n(t) \rangle$, the expected number of cells at time t. Is the answer what you'd expect?