Efficiency Wages

It is increasing practice in modern times for firms to pay "efficiency wages" to keep their employees happy enough that they are active on the job. This problem explores a simple model that explains the reasoning behind efficiency wages.

Let's begin by looking at the workers. The state space of any given worker can be approximated as

$$\Omega_{\text{worker}} = \{\mathrm{A}, \mathrm{S}, \mathrm{U}\},$$

where A means the worker is actively working, S means the worker is shirking (not working, but employed), and U means the worker is unemployed. Let us approximate that an unemployed worker will gain a job as a (stopped) Poisson process with rate λ, an A worker will lose their job with rate μ, and a S worker will lose their job with rate $\mu + \nu$.

Workers make decisions based on utility. Given the path $\omega(t)$ of a given worker through state space, we can define

$$U[\omega] = \int_0^\infty \mathrm{d}t\; u(\omega(t))\mathrm{e}^{-\rho t}$$

where the random variable $u(t)$ is the utility rate per unit time that the worker has in state ω, which we assume takes the following simple form:

$$\begin{aligned} u(\mathrm{A}) &= w - e \\ u(\mathrm{S}) &= w \\ u(\mathrm{U}) &= 0 \end{aligned}$$

where e is a measure of the effort exerted by workers, and w is the wage they are paid. Let us also define the function

$$V(\omega_0) \equiv \langle U[\omega(t)] | \omega(0) = \omega_0 \rangle$$

for each $\omega_0 \in \Omega_{\text{worker}}$.

(a) Explain why $V(\omega_0)$ is t-independent, and

$$V(\omega_0) = \left\langle \int_0^\tau \mathrm{d}t\; u(\omega(t))\mathrm{e}^{-\rho t} + V(\omega(\tau))\mathrm{e}^{-\rho\tau} \middle| \omega(0) = \omega_0 \right\rangle.$$

(b) Take the derivative of this expression with respect to τ, at $\tau = 0$, given each possible initial state. This will allow you to compute the function V.

Now, let's look at things from the employer's perspective. Assume that there are N firms, and each firm employs about l workers on average. The economy has L_{T} workers in total. Firms will choose w so that $V(\mathrm{A})$ is not smaller than $V(\mathrm{S})$: thus, it expects all its workers to be in the A state.

(c) Show that the firm will pick wages so that

$$V(\mathrm{A}) = V(\mathrm{U}) + \frac{e}{\nu}.$$

(d) Express λ in terms of μ, N, l and L_{T}.

(e) Determine w in terms of e, μ, ρ, N, l and L_{T}.

The employer's profit is given by

$$\pi = f(el) - wl$$

where f is a monotonically increasing, concave down function.

(f) Assuming the employer is profit maximizing, find the condition for how many workers the firm will hire.

(g) Sketch a plot of l vs. w along with the regions where the firm is profit maximizing, as well as where the workers are not shirking.

(h) Determine whether an increase in each of the following variables will cause w to rise: L_{T}, ν, e, N.